Numerical analysis of displacements of a diaphragm wall

M.Mitew
Warsaw University of Technology, Warsaw, Poland

ABSTRACT: In the paper, numerical analysis and parameter study have been presented taking into account two calculation methods, based on different constitutive soil models. Numerical back analysis has been divided into two parts. The first part considers the analysis performed using “Geo4-Sheeting analysis” and Rido - software, employing the method of dependent pressures, in which magnitudes of pressures acting upon a structure depend on deformation of the structure. Due to its simplicity, the method is widely used for the design purposes. For this reason, it was chosen for wider discussion in the paper. The second part includes numerical analysis performed using finite element method – software: Geo4-FEM and Plaxis 7.2. In the last part of the paper the results of numerical analysis have been compared to the results of in-situ measurements, over-all discussion has been performed and conclusions have been presented.

1 INTRODUCTION

The paper presents the results and discussion under the wide scope of numerical back analysis of the displacements of a diaphragm wall. The case study considered within this paper is 3 level underground structure executed within 80 cm thick and 14,5 m deep diaphragm wall. The cross-section including geotechnical conditions as well as the stages of construction, chosen for the calculation is shown in Figure 1.

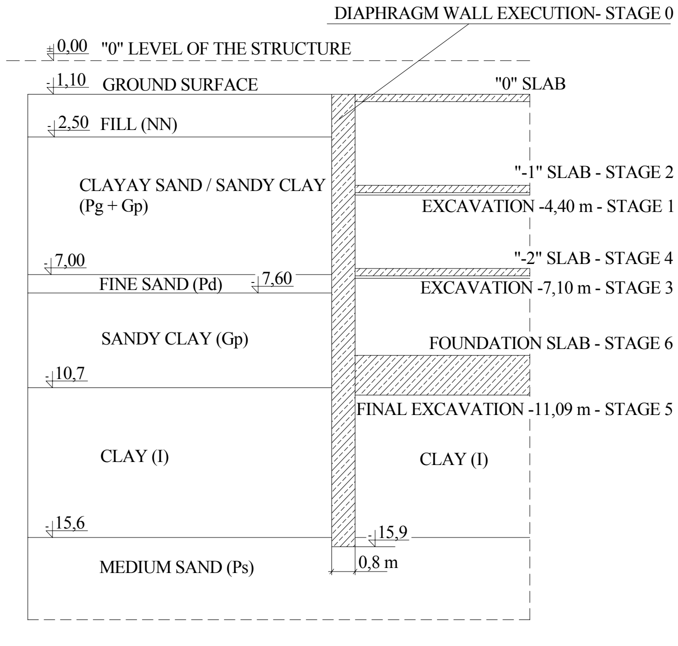

Figure 1. The cross-section

Full Milan method has been applied to provide the stability of the walls of the excavation (Fig.2). The excavation was executed in the following stages:

Stage 1 – excavation to the depth of 4.40 m,
Stage 2 – casting of “-1” slab,
Stage 3 – excavation to the depth of 7.10 m,
Stage 4 – casting of “-2” slab,
Stage 5 – excavation to the final depth - 14.60 m,
Stage 6 – casting the ground slab;

During the construction, continuous monitoring of diaphragm wall displacements has been carried out using automatic inclinometer chain - Geokon Vibrating Wire In-Place Inclinometer, designed for long-term monitoring of deformations of structures (Siemińska-Lewandowska & Mitew 2002).

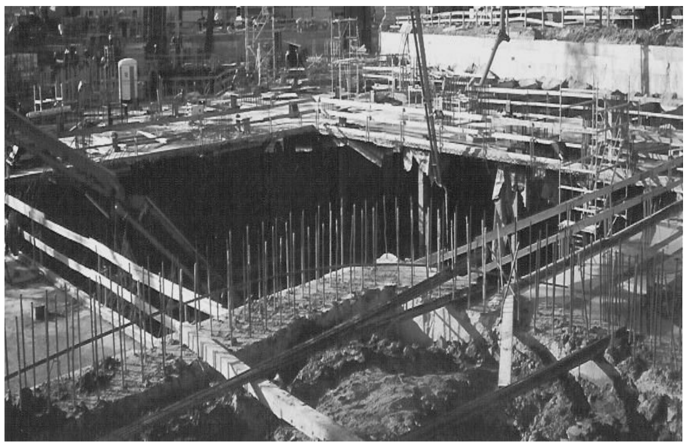

Figure 2. The site (deep excavation) presented in the paper

The results of in-situ measurements have been analysed, the displacements of the diaphragm wall at each construction stage have been determined. Specified displacement values have been used in the back analysis in order to compare it to the results of all numerical analysis.

2 THE METHOD OF DEPENDING PRESSURES

2.1 *Calculation model*

For modelling of soil the subgrade reaction modulus method (dependent pressures method) uses one-parameter Winkler analogue subsoil model (Winkler 1867). The soil-wall contact is replaced by a system of independent elastic supports of stiffness k_h.
The wall is treated as an elastic beam of a unit width and the value of the horizontal, elastic soil reaction at examined point is directly proportional to horizontal wall displacement at the same point.

$$p_z = k_h y \tag{1}$$

and

$$y = y(z) \tag{2}$$

In case of the discussed method, the key question is the determination of k_h modulus, which cannot be identified with the coefficient defined by Winkler. The methods of k_h parameter determination are presented below.

2.2 *Analytic and empirical methods of determination of k_h modulus*

Since k_h parameter (subgrade reaction modulus) is not a physical value defining the soil, but a calculation parameter depending on the stiffness of the wall (EI), wall geometry (ratio of excavation depth to the depth of the wall below its bottom) and soil conditions. There is no possibility to define k_h using in-situ methods. The majority of k_h determination methods make use of displacement calculations of a rigid diaphragm wall acting in condition of passive earth pressure.
Many methods are known in literature (Siemińska-Lewandowska 2001) for determination of the k_h modulus based on the classical theory of elasticity or on empirical investigation, particularly those making use of the results of pressuremeter investigations. In this paper three of them shall be discussed: Terzaghi's method, Chadeisson and Monnet as well as Menard and Bourdon methods, characteristic by separate approach to the problem.

2.2.1 *The method of Terzaghi*

Terzaghi's method is based on the principles of classical theory of elasticity (Terzaghi 1955). In case of non-cohesive soils according to Terzaghi's assumptions, the value of k_h at given depth z depends on wall dimension perpendicular to displacement, weight of soil and density index of the soil.

$$k_h = \frac{p}{y} = \frac{A\gamma}{1{,}35}\frac{z}{B} = m_H z = n_H \frac{z}{B} \tag{3}$$

B - wall dimension perpendicular to its displacement,
n_H - constant depending on the density index of the soil; the values of n_H are shown in Table 1.

Table 1. Values of n_H [kN/m^3] for a wall of width B=1m, cast in sand

Density index	Loose	Medium dense	Dense
Dry and moist sand	2230	6700	17890
Wet sand	1280	4470	10860

Equation (3) can be applied for non-cohesive soils. In case of cohesive soils the value of k_{H1} becomes the same as the value of parameter k_{SI} established for a beam resting on horizontal surface of the same kind of soil. Based on expression (3), the value of k_h for wall of a unit width can be expressed by Equation (4):

$$k_h = \frac{1}{B}k_{H1} = \frac{1}{B}k_{S1} = \frac{1}{1{,}5B}\bar{k}_{S1} \tag{4}$$

where:
B – width of wall,
k_{SI} - constant depending on the liquidity index of the soil.
Assuming B=1m, the values of k_h can be determined depending on the liquidity index (Tab. 2).

Table 2. The k_h [kN/m^3] for 1m wide wall, cast in cohesive soil

State of clay	Medium	Stiff	Very stiff
Dry and moist sand	16000	32000	63900

2.2.2 *The method of Monnet*

The goal of this empirical method is the evaluation of the magnitude of displacement necessary to mobilise the limit passive pressure. R. Chadeisson (1961) based on many years of investigations in constructing of diaphragm walls, 60cm and 80cm thick, in varied geotechnical conditions, determined the value of k_h depending on the shear strength of soil (Coulomb - Mohr criterion) i.e. c' and ϕ' parameters, introducing into calculations the stiffness of the wall. Developed by Chadeisson and later simplified by Monnet (1994) the formula for determining the value of subgrade reaction modulus k_h for given subsoil has the following form:

$$k_h = \left[20EI \left(\frac{K_p \gamma (1 - K_0 / K_p)}{dr_0} \right)^4 \right]^{\frac{1}{5}} + A_p c' \frac{th(c'/c_0)}{dr_0} \quad (5)$$

where:
γ - specific gravity of soil,
K_P - passive pressure coefficient,
K_0 - pressure coefficient at rest,
dr_0 - characteristic displacement (0.015m),
c' - effective cohesion,
A_P - coefficient allowing for soil cohesion,
c_0 - 30 kPa.

Substituting the values of parameters K_P, K_0, γ, c' into above expression and assuming wall thickness 80 cm ($E=2x10^7$ *kPa*) the chart shown in Figure 3 was obtained, serving to evaluate k_h on the basis of parameters c' and ϕ'.

2.2.3 *The method of Menard and Bourdon*

Menard and Bourdon (1964) made the first approach towards empirical determination of the value of subgrade reaction modulus taking advantage of pressuremeter investigation results. The method developed by them was complemented in later years by Balay (1984), Gigan (1984) and Schmitt (1995). On the base of pressuremeter tests results in the surroundings of retaining walls, Menard and Bourdon determined the relationship between kh and the pressuremeter modulus by the following expression.

$$k_h = \left[\frac{1}{E_M} \left[\frac{\alpha a}{2} + 0.133 (9a)^\alpha \right] \right]^{-1} \quad (6)$$

where:
E_M - pressuremeter modulus of soil,
α - rheological soil coefficient, assumed:
1/3 in non-cohesive soils,
1/2 in silts,
2/3 in cohesive soils,
a [m] - height, within which soil is acting in passive pressure, defined by Menard as 2/3 amount of the penetration of the wall below the final bottom of the excavation.

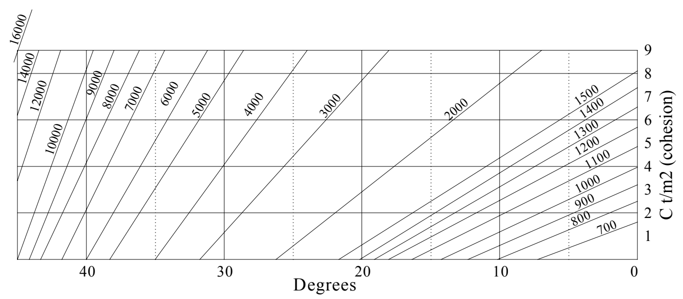

Figure 3. The chart of Chadeisson for the evaluation of k_h basing on c' and ϕ' values

3 APPLICATION OF THE METHOD OF DEPENDENT PRESSURES IN DIAPHRAGM WALL ANALYSIS

Three calculation sequences were made for a chosen characteristic cross-section, leaving basic geotechnical parameters (defined after the geological report) without change, but varying subgrade reaction modulus k_h, defined according to the methods discussed in paragraph 2. Parameters of individual geological layers are compiled in Table 3 together with the suitable moduli k_h defined on the basis of Terzaghi's, Chadeisson-Monnet and Menard-Bourdon equations. The results of each sequence of calculations are compiled in Table 4.

Table 3. Geotechnical parameters of individual soil layers

	I_D / I_L	γ kN/m^3	c_u kPa	φ_u [°]	k_h [kN/m^3]		
					Terzaghi	Chaidesson -Monnet	Menard-Bourdon
1	-	19.0	0	22	2230	16000	6000
2	0.27	21.8	7	27	8000	20500	4100
3	0.60	20.2	0	34	4470	37000	20200
4	0.00	22.5	15	28	32000	27000	14400
5	0.10	20.0	25	16	16000	15000	7500
6	0.70	20.7	0	36	10860	43000	41500

Table 4. Comparison of results of analysis (Geo4-Sheeting analysis, Rido)

	Terzaghi	Chadeisson -Monnet	Menard-Bourdon	Measured value
Max disp. [mm]	9.2	9.7	18.9	12.3

4 FEM ANALYSIS OF THE CASE

Numerical analysis of the structural model using Finite Element Method (FEM) includes - apart from the diaphragm wall under examination - also the interacting soil and objects in wall environment. The choice of the soil constitutive model is the basic element of FEM analysis. Substantial number of models can be mentioned (Gryczmański 1995), from which the most often applied in geotechnics are elastic-ideal plastic models with associate law of flow and isotropic plasticity surface (e.g. Coulomb - Mohr, Tresca, Huber - Mieses - Hencky, Drucker - Prager) and elastic-plastic models with isotropic strain hardening of volumetric kind (e.g. Cam-Clay, Modified Cam-Clay). Depending on the soil model adopted, different parameters are needed during analysis. In engineering practice the most popular is the elastic-ideal plastic model with Coulomb-Mohr plasticity surface, because of its simplicity and small number of model parameters (ϕ, c, E, ν), which can be determined on the basis of laboratory investigation, or in-situ.

Finite element plain strain analysis has been carried out using GEO4-FEM and PLAXIS version 7.2 software. Due to the fact, that sophisticated soil parameters were not available - simple Coulomb-Mohr constitutive soil model was chosen for modelling the soil body. The diaphragm walls, as well as supporting slabs were modelled as beam elements (2D elements). Contact elements have been applied for modelling the interaction between the soil and the structure. Four calculation sequences have been performed varying the stiffness (modulus of elasticity - E) of soil layers. In the Table 5, calculation sequences are called respectively: FEM 1 – stiffness determined according to Polish Code, FEM 2 – stiffness determined basing on literature recommendation, FEM 3 - stiffness determined basing on geotechnical report recommendation, FEM 4 - stiffness determined basing on pressuremeter investigation results. In each case described above "staged construction" analysis have been carried out in order to obtain results (displacements) in all construction phases. FEM model mesh (Fig. 4), generated automatically, consisted of 773 - 6 or 15 nodes elements, 1741 nodes and 2319 stress points. The theoretical displacements resulting from all calculation cycles have been compared to the results of in-situ displacements measurements. Detailed results are presented in Table 5.

Table 5. Comparison of results of analysis (Geo4-FEM, Plaxis v7.2)

	FEM 1	FEM 2	FEM 3	FEM 4	Measured value
Max disp. [mm]	12.05	12.76	11.71	11.10	12.3

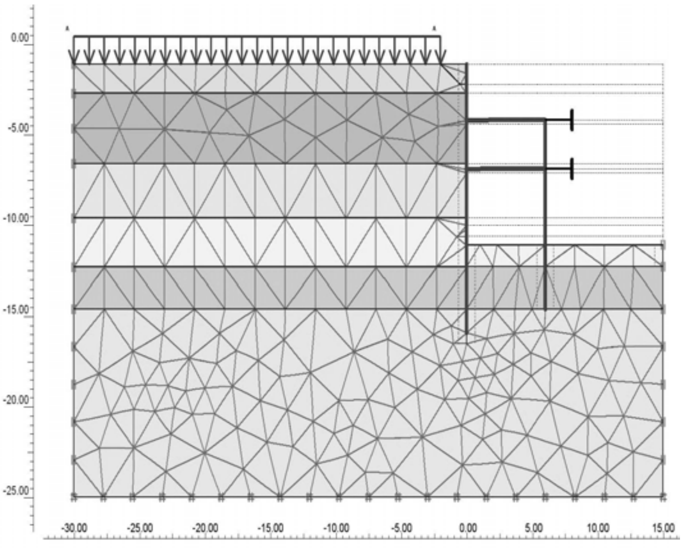

Figure 4. Numerical model

5 SUMMARY

In total seven calculation sequences have been performed using two types of software: based on the method of depending pressures and the other - using Finite Element Method (FEM). Results of static analysis of the case are compared in Table 6.

Table 6. Comparison of results of all analysis (Geo4-Sheeting analysis, Rido, Geo 4-FEM, Plaxis v7.2)

	Dependent pressures method			Finite Element Method				Measured value
	Terzaghi	Chadeisson -Monnet	Menard-Bourdon	FEM 1	FEM 2	FEM 3	FEM 4	
Max disp. [mm]	9.2	9.7	18.9	12.1	12.8	11.7	11.1	12.3

The results obtained in static analysis using the dependent pressures method prove the significant influence of the value of subgrade reaction modulus k_h on determined theoretical displacements (as well as on internal forces). K_h modulus, determined using various methods for individual geotechnical layers is varying within the range:

4100 - 20500 kN/m^3 - layer 2 - clayey sand/ sandy clay,
4470 - 37000 kN/m^3 - layer 3 - fine sand,
14400 - 32000 kN/m^3 - layer 4 - sandy clay,
7500 - 16000 kN/m^3 - layer 5 - clay,
10860 - 43000 kN/m^3 - layer 6 - medium sand.

So large scattering of values results in visible differences in obtained results. It can be observed, that the displacements determined using theoretical methods of specifying k_h modulus (Terzaghi, Monnet) are similar, despite great differences in module values for individual layers. Results obtained on the basis of empirical methods (Menard) differ much from those based on theoretical methods, but are nearer to real values (from the point of view of safety).

Maximum theoretical displacements of the wall estimated in all FEM analysis, in general, are very close to the value of maximum real displacement measured during construction. The approximate compatibility of the results of measurements and calculations has been observed in the top part of the wall in all calculation series and in all construction stages. All calculation sequences showed a significant diaphragm wall displacement towards the excavation in the span above the foundation slab in last two construction phases, which has not been observed in-situ. That may be due to a great, not realistic, relaxation of the bottom of the excavation estimated in the FEM calculations. Relatively best results have been obtained when the soil parameters were based on the results of pressuremeter investigations. Further analysis of the case will be performed.

Taking into account the difficulties in assessing the parameters of the soils as well as the great discrepancy of obtained results, it should be stressed that regardless of the method of static analysis, close observation of the real diaphragm wall displacements is a necessary item in construction process.

REFERENCES

Balay, J. 1984. Recommandations pour le choix des paramètres de calcul des écrans de soutènement par la méthode aux modules de réaction. *Note d'information technique*. Paris : LCPC

Chadeisson, R. 1961. Parois continues moulées dans le sol, *Proceedings of the 5th European Conference on Soil Mechanics and Foundation Engineering*. Paris : Dunod, Vol. 2, p. 563-568,

Gigan, J.P. 1984. Expérimentation d'un rideau en palplanches ancré par tirants actifs. *Bulletin de liason des Laboratoires des Ponts et Chaussées* No 129, p. 5 –20

Gryczmański, M. 1995. Wprowadzenie do opisu sprężysto-plastycznych modeli gruntów. *Wyd. KILiW PAN, IPPT PAN Studia z zakresu inżynierii*, No. 40. p.156

Ménard, L. & Bourdon, G. & Houy, A. 1964. Étude expérimental de l'encastrement d'un rideau en fonction des charactéristiques pressiométriques du sol de fondation. *Soil. Sols* No. 9

Monnet, A. 1994. Module de réaction, coefficient de décompression, au sujet des paramètres utilisés dans la méthode de calcul élastoplastique. *Revue Française de Géotechnique* 66. p. 67-72

Schmitt, P. 1995. Méthode empirique d'évaluation du coefficient de réaction du sol vis à vis des ouvrages de soutènement souples. *Revue Française de Géotechnique 71*. p. 3-10

Siemińska-Lewandowska, A. 2001. Przemieszczenia kotwionych ścian szczelinowych, *Wyd. Oficyna Wydawnicza Politechniki Warszawskiej, Prace naukowe*, *Budownictwo* z. 139

Siemińska-Lewandowska, A. & Mitew, M. 2002. Diaphragm wall monitoring report. *Geokonstrukcja Sp. z o.o*. Warsaw

Terzaghi, K. 1955. Evaluation of coefficients of subgrade reactions*. Géotechnique*. Vol. 4

Winkler, E. 1867. Die Lehre von der Elastizitat und Festigkeit. *Dominicus*. Prague